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The influence of outflows on the $1/f$-like luminosity fluctuations

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ABSTRACT

In accretion systems, outflows may have significant influence on the luminosity fluctuations. In this paper, following the Lyubarskii's general scheme, we revisit the power spectral density of luminosity fluctuations by taking into account the role of outflows. Our analysis is based on the assumption that the coupling between the local outflow and inflow is weak on the accretion rate fluctuations. We find that, for the inflow mass accretion rate $\dot{M} \propto r^s$, the power spectrum of flicker noise component will present a power-law distribution $p(f) \propto f^{-(1+4s/3)}$ for advection-dominated flows. We also obtain descriptions of $p(f)$ for both standard thin discs and neutrino-cooled discs, which show that the power-law index of a neutrino-cooled disc is generally larger than that of a photon-cooled disc. Furthermore, the obtained relationship between $p(f)$ and s indicates the possibility of evaluating the strength of outflows by the power spectrum in X-ray binaries and gamma-ray bursts. In addition, we discuss the possible influence of the outflow-inflow coupling on our results.

Key words: accretion, accretion discs - X-rays: binaries; ISM: jets and outflows

1 INTRODUCTION

The emission of Galactic Black Hole Binaries (BHBs) and active galactic nuclei (AGN) displays a significant aperiodic variability on a broad range of time-scales. The Power Spectral Density (PSD) of such variability is generally modeled with a power law, $p(f) \propto f^{-\beta}$, where $p(f)$ is the power at frequency f, and the power-law index β keeps a constant in a certain range of f, but changes among different ranges. At high frequencies, the PSDs of both BHBs and AGN present a steep slope with $\beta \sim 2$. On the contrary, below a break frequency, typically at a few Hz for BHBs, they flatten to a slope with $\beta \sim 1$, representing the flicker noise (see King et al. 2004 and references therein).

Several models have been proposed in order to understand this nearly featureless character of power spectra. The so-called "shot noise models" (Terrell 1972) attempted to describe the light curves as a series of independent overlapping shots with specific time-scales, amplitudes, and occurence rates. Due to lack of physical picture in this scenario, various physically motivated ideas have been put forward subsequently, such as the fluctuations of hydrodynamic or magnetohydrodynamic turbulence (Nowak & Wagoner 1995; Hawley & Krolik 2001), magnetic flares or density fluctuation in the corona (Galeev et al. 1995; Poutanen & Fabian 1995; Goosmann et al. 2006; Kawanaka et al. 2008), and Lyubarskii's general scheme (Lyubarskii 1997; King et al. 2004). In the Lyubarskii's scheme, it was noted that any variation of accretion rate, which is caused by small amplitude variations in the viscosity, would induce a variation in the accretion rate at the inner radius of the disc, where most of the energy is released. Moreover, observations showed that the variability is non-linear and the rms variability is proportional to the average flux over a wide range of time-scales (e.g., Uttley & McHardy 2001; Uttley et al. 2005; Gleissner et al. 2004). It indicates that the short time-scale variations are modulated by the longer time-scales, which favors the Lyubarskii's scheme.

However, the observed power spectra often deviate from the form f^{-1}. For example, the PSD of Cyg X-1 is well described with the form f^{-1} in the soft state, however, exhibits the form $f^{-1.3}$ in the hard state (Gilfanov 2010). In particular, it is shown that the power-law index is around $0.8 - 1.3$ both in the soft state of BHBs and in narrow-line Seyfert 1 galaxies (Janiuk & Czerny 2007). Such a dispersion of the power-law index reveals that there must exist some other mechanism. A radius-dependent amplitudes of α fluctuations may help to alleviate the discrepancy between theories and observations. However, it remains unclear why the fluctuations of α should have a strong radius-dependent form.

In the present paper, we will take into account another mechanism, outflows, which is a popular phenomenon in accretion systems and has strong observational evidence.

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One of the best examples comes from Sgr A*, whose center harbors a supermassive black hole surrounded by an accretion flow that is likely to be in the form of the advection-dominated accretion flow (ADAF, Narayan & Yi 1994). Radio polarization observations constrain the accretion rate in the innermost region is nearly two orders of magnitude lower than that measured at the Bondi radius (e.g., Marrone et al. 2006), which indicates that intense outflows may present in this system. Besides, the absorption lines from highly ionized elements, which have been detected in the X-ray spectrum of some microquasars such as GRO J1655-40 (Ueda et al. 1998; Yamaoka et al. 2001; Miller et al. 2006), GRS 1915+105 (Kotani et al. 2000; Lee et al. 2002) and Atoll sources (see e.g. the review by Díaz Trigo et al. 2006 and references therein), also indicate the existence of outflows. On the other hand, Jiao & Wu (2011) found that outflows generally exist in accretion discs no matter that the flow is advection-dominated such as the slim disc (Abramowicz et al. 1988) and the ADAF, or is radiation-dominated such as the standard thin disc (Shakura & Sunyaev 1973). In particular, for the three types of advection-dominated flows: ADAFs (gas internal energy dominant), slim discs (trapped photon energy dominant), and hyper-accretion discs (trapped neutrino energy dominant), outflows may be significantly strong due to positive Bernoulli parameters (e.g., Narayan et al. 1997; Liu et al. 2011) or the large radiation pressure (e.g., Gu & Lu 2007). Furthermore, outflows have generally been found in many simulation works (e.g., Ohsuga & Mineshige 2011 and references therein).

In the Lyubarskii's scheme, the power spectrum of luminosity fluctuations is sensitive to the varying mass accretion rate, thus we expect that outflows may have significant effects on the power-law index. The paper is organized as follows. In Section 2, we investigate the fluctuation power spectrum under a radius-dependent accretion rate following the method of Lyubarskii. The potential application of our results to observations is discussed in Section 3.

2 LUMINOSITY FLUCTUATIONS WITH A RADIUS-DEPENDENT ACCRETION RATE

2.1 Evolution equations of the fluctuations

In the present work, following the Lyubarskii's general scheme, we revisit the power spectral density of luminosity fluctuations by considering a radius-dependent accretion rate. The relevant processes are the conservation of mass and angular momentum. With a radius-dependent accretion rate, these two processes are described as follows:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{2\pi r}\left(\frac{\partial \dot{M}}{\partial r} - \Phi\right), \tag{1}$$

$$\frac{\partial(\dot{M}\Omega r^2)}{\partial r} = -\frac{\partial}{\partial r}(2\pi r^2 \mathrm{T}_{r\varphi}) + \Phi l_\Phi, \tag{2}$$

where Σ, $\dot{M}$, Ω and $\mathrm{T}_{r\varphi}$ are the surface density, accretion rate, angular velocity and $r\varphi$ component of the stress tensor at the radius r, respectively. Φ is the change of accretion rate over r in the stationary accretion state, i.e. $\Phi = \partial\dot{M}/\partial r$, and l_Φ is the angular momentum of Φ.

With $\Omega r^2 = l_{\rm in}$, $F(= -\mathrm{T}_{r\varphi} r^2) = 0$ at the inner radius $r_{\rm in}$, we can obtain

$$\begin{aligned} F &= \frac{1}{2\pi}[\dot{M}\Omega r^2 - \dot{M}_{\rm in} l_{\rm in} - \int_{r_{\rm in}}^{r} \Phi l_\Phi dr] \\ &= \frac{\dot{M}\Omega r^2}{2\pi}[1 - (\dot{M}_{\rm in} l_{\rm in} + \int_{r_{\rm in}}^{r} \Phi l_\Phi dr)/(\dot{M}\Omega r^2)]. \end{aligned}$$

With the following definitions:

$$h = \Omega r^2, \tag{3}$$

$$Q = 1 - (\dot{M}_{\rm in} l_{\rm in} + \int_{r_{\rm in}}^{r} \Phi l_\Phi dr)/(\dot{M}\Omega r^2), \tag{4}$$

the above description of F can be simplified as

$$F = \frac{\dot{M}h}{2\pi}Q.$$

For standard thin discs and ADAFs, $\Omega \propto \Omega_{\rm K} = \sqrt{GM/r^3}$, and thus $h \propto r^{1/2}$. We assume $h = b\sqrt{GMr}$ in this paper, where b is a constant. If the accretion rate has weak dependence on the radius and $l_\Phi \propto \Omega r^2 \propto \sqrt{r}$, the value of Q would remain nearly constant. In order to simplify the problem, we assume Q to be constant in our analysis. The validity of this assumption is discussed in Appendix A.

With $\chi = (1/2\pi)\int \Phi(l_\Phi - \Omega r^2)dr$, equation (2) becomes

$$\dot{M} = -\frac{\partial[2\pi(F+\chi)]}{\partial h}. \tag{5}$$

Substituting the above equation into the equation (1), we have

$$\frac{\partial \Sigma}{\partial t} = \frac{b^4(GM)^2}{2h^3}\frac{\partial^2(F+\chi)}{\partial h^2} - \frac{b^2 GM}{2\pi h^2}\Phi. \tag{6}$$

The relationship between Σ and F can be deduced with α-prescription and the local balance between the heating and the cooling in the disc. The formula is similar to that with a constant accretion rate (Filipov L.G. 1984; Lyubarskii 1987; Narayan & Yi 1994), i.e.

$$\Sigma = \frac{b^4(GM)^2 F^{1-m}}{2(1-m)Dh^{3-n}}, \tag{7}$$

where the exponents m and n are determined by the disc model, which are discussed in §2.2, and D is a function of α. It should be noted that the analysis presented here is not strict for ADAFs. A stringent analysis for the ADAFs may refer to Appendix B.

Substituting equation (7) into equation (6), we have

$$\begin{aligned} \frac{\partial F}{\partial t} = \frac{2DF^m}{b^4(GM)^2 h^{n-3}}[&\frac{b^4(GM)^2}{2h^3}\frac{\partial^2(F+\chi)}{\partial h^2} - \\ &\frac{b^2 GM}{2\pi h^2}\Phi] + \frac{F}{(1-m)D}\frac{\partial D}{\partial t}. \end{aligned} \tag{8}$$

By assuming $\alpha = \alpha_0[1 + \overline{\beta}(t,r)]$, where $\overline{\beta}(t,r) (\ll 1)$ is decorrelated at different radial scales and its correlation time-scale is of the order of the local viscous time-scale, the disturbed quantities D and F are

$$D = D_0(1 + \eta\overline{\beta}), \quad F = (\frac{\dot{M}_0 h}{2\pi})Q + \psi,$$

where η is defined as $D \propto \alpha^\eta$ and the subscript 0 denotes unperturbed quantities. Substituting the above equations into

equation (8), and including the following stationary condition

$$\frac{b^4(GM)^2}{2h^3}\frac{\partial^2}{\partial h^2}(\frac{\dot{M}_0 h}{2\pi}Q+\chi)-\frac{b^2GM}{2\pi h^2}\Phi=0,$$

we have,

$$\frac{\partial\psi}{\partial t}=\frac{D_0Q^m}{h^{n-m}}(\frac{\dot{M}_0}{2\pi})^m\frac{\partial^2\psi}{\partial h^2}+\frac{\dot{M}_0 hQ}{2\pi(1-m)}\frac{\partial\eta\overline{\beta}}{\partial t}. \tag{9}$$

Here, we assume that the coupling between the local outflow and inflow is weak on the accretion rate fluctuations. A simple discussion on this issue is presented in §2.4.

2.2 The analysis of luminosity fluctuations

Assuming the radius-dependent accretion rate is

$$\dot{M}_0 = \dot{M}_{0,\,\mathrm{in}}(\frac{r}{r_{\mathrm{in}}})^s, \tag{10}$$

where $\dot{M}_{0,\,\mathrm{in}}$ is the accretion rate at the inner radius r_{in}, we have

$$\frac{\partial\psi}{\partial t}=\frac{C}{h^{n-m-2sm}}(\frac{\dot{M}_{0,\,\mathrm{in}}}{2\pi})^m\frac{\partial^2\psi}{\partial h^2}+\frac{\dot{M}_{0,\,\mathrm{in}}hQ\eta}{2\pi(1-m)}\frac{\partial}{\partial t}[\overline{\beta}(\frac{h^2}{b^2GMr_{\mathrm{in}}})^s] \tag{11}$$

and $C=D_0Q^m/(GMr_{\mathrm{in}}b^2)^{sm}$. This is a linear diffusion equation with a radius-dependent accretion rate. For $s=0$, i.e. a constant accretion rate in the system, we have $Q=1$ and equation (11) is reduced to

$$\frac{\partial\psi}{\partial t}=\frac{D_0}{h^{n-m}}(\frac{\dot{M}_{0,\,\mathrm{in}}}{2\pi})^m\frac{\partial^2\psi}{\partial h^2}+\frac{\dot{M}_{0,\,\mathrm{in}}h\eta}{2\pi(1-m)}\frac{\partial\overline{\beta}}{\partial t}, \tag{12}$$

which is the exact form of equation (9) in Lyubarskii (1997).

In general, the solution of equation (11) is (Lyubarskii 1997, Lynden-Bell & Pringle 1974)

$$\psi(t,x)-\psi(0,x)=\frac{\dot{M}_{0,\,\mathrm{in}}\eta Q\kappa^2x^l}{4\pi(1-m)}\int_0^t dt'\int_0^\infty dx_1\frac{x_1^{l+1}}{t-t'}$$
$$\exp[-\frac{(x^2+x_1^2)\kappa^2}{4(t-t')}]I_l[\frac{\kappa^2xx_1}{2(t-t')}]\frac{\partial}{\partial t'}[\overline{\beta}(t',x_1)\frac{x_1^{4ls}}{(b^2GMr_{\mathrm{in}})^s}],$$

where

$$l=\frac{1}{2+n-m-2sm}\ , \tag{13}$$

$$x=h^{1/2l},\ 4(\frac{l}{\kappa})^2=C(\frac{\dot{M}_{0,\,\mathrm{in}}}{2\pi})^m.$$

From equation (2), the accretion rate $\dot{M}(t,x)$ is

$$\dot{M}(t,x)-\dot{M}(0,x)=$$
$$\int_0^t\int_0^\infty G(t,x;t',x_1)\frac{\partial}{\partial t'}[\overline{\beta}(t',x_1)\frac{x_1^{4ls}}{(b^2GMr_{\mathrm{in}})^s}]dt'dx_1,$$

where

$$G(t,x;t',x_1)=\frac{\eta Q\kappa^4x^{1-l}x_1^{l+1}}{8l(1-m)(t-t')^2}\exp[-\frac{(x^2+x_1^2)\kappa^2}{4(t-t')}]$$
$$\{x_1I_{l-1}[\frac{\kappa^2xx_1}{2(t-t')}]-xI_l[\frac{\kappa^2xx_1}{2(t-t')}]\},$$

and $I_\nu(z)$ is the Bessel function of the imaginary argument. Based on the above equation, the power spectrum of $\dot{M}(t,x)$ can be obtained following a complex calculation as presented in Section 4 of Lyubarskii (1997). Here, we directly present the result of the power spectrum and focus on the effects of outflows on the luminosity fluctuations. If $\sqrt{<(\overline{\beta})^2>}\propto\ r^\xi$, the power spectrum $p(f)$ of $\dot{M}(t,x)$ is

$$p(f)\propto f^{-[1+4l(\xi+s)]}, \tag{14}$$

which indicates the power-law index $\beta=1+4l(\xi+s)$. In this work we ignore α fluctuations, i.e. $\xi=0$, and then the expression of β will be reduced to

$$\beta=1+4ls. \tag{15}$$

For an advection-dominated flow, we have $m=0$, $n=1$, and $D\propto\alpha$ (Narayan & Yi 1994) for equation (7), thus equation (13) indicates $l=1/3$ and therefore β can be simplified as

$$\beta=1+\frac{4}{3}s. \tag{16}$$

We would like to stress that the above formula should be valid for all the three types of advection-dominated flows mentioned in the first section.

For a standard thin disc, it is well-known that there exist three regions according to different dominant mechanisms for opacity and pressure. We have $m=0.3$, $n=0.8$, and $D\propto\alpha^{0.8}$ for the outer region ($p\sim p_{\mathrm{gas}}$, $\kappa\sim\kappa_{\mathrm{ff}}$), $m=0.4$, $n=1.2$, and $D\propto\alpha^{0.8}$ for the middle region ($p\sim p_{\mathrm{gas}}$, $\kappa\sim\kappa_{\mathrm{es}}$), and $m=2$, $n=7$, and $D\propto\alpha$ for the inner region ($p\sim p_{\mathrm{rad}}$, $\kappa\sim\kappa_{\mathrm{es}}$), thus we easily obtain the following from equations (13) and (15):

$$\beta=1+\frac{40}{25-6s}s \tag{17}$$

for the outer region,

$$\beta=1+\frac{10}{7-2s}s \tag{18}$$

for the middle region, and

$$\beta=1+\frac{4}{7-4s}s \tag{19}$$

for the inner region. We would point out that outflows may be negligible in standard thin discs expect for the inner region, which is radiation pressure dominated and may suffer the thermal instability. Outflows in the inner region may be significantly stronger than that in the other two regions due to the thermal instability. However, it remains under controversy whether the inner region is indeed thermally unstable or not (e.g., Hirose et al. 2009), and many mechanisms have recently been proposed to suppress the instability (e.g., Zheng et al. 2011; Lin et al. 2011; Ciesielski et al. 2011).

Apart from the above mentioned photon-radiation-dominated flows, neutrino-dominated accretion flows (NDAFs) have also been widely studied (e.g., Popham et al. 1999; Narayan et al. 2001; Gu et al. 2006; Chen & Beloborodov 2007; Liu et al. 2007), which may account for the central engine of gamma-ray bursts (GRBs, e.g., Narayan et al. 1992). The outer region of NDAFs may be advection-dominated since neutrino cooling cannot balance the viscous heating due to low temperature and density. This region can be regarded as an extension of slim discs (e.g., Liu et al. 2008). On the other hand, for high mass accretion rates such as $\dot{m}\gtrsim 1M_\odot\ \mathrm{s}^{-1}$, the inner

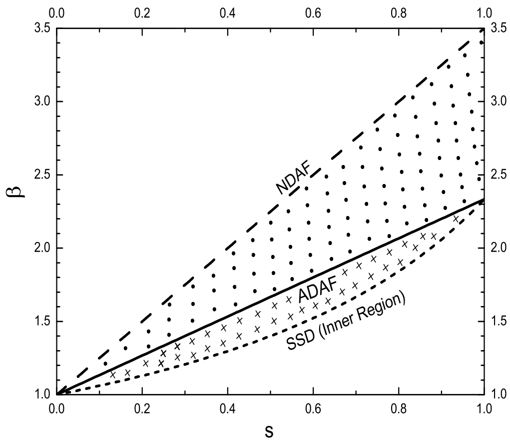

Figure 1. Variation of β with s for three types of accretion models: radiation-pressure-supported standard thin discs (the short dashed line), ADAFs (the solid line), and NDAFs (the long dashed line). The region of photon-cooled discs and that of neutrino-cooled discs are marked by the cross symbol and the dot symbol, respectively.

region may also become advection dominant due to the large optical depth for neutrinos (e.g., Di Matteo et al. 2002; Gu et al. 2006). Outflows may occur in NDAFs in particular for these two advection-dominated regions owing to positive Bernoulli parameters (e.g., Liu et al. 2011).

For NDAFs, with the balance between the cooling rate by pair capture and the energy dissipation rate per unit volume (Popham et al. 1999, Eqs. 5.4-5.5), we have $m = 0$, $n = -0.4$, and $D \propto \alpha^{1.2}$, thus equations (13) and (15) directly give the form of β:

$$\beta = 1 + \frac{5}{2}s. \tag{20}$$

The relationship between β and s for all the mentioned accretion models are presented in Figure 1, where the short dashed, solid, and long dashed lines correspond to the inner region of standard thin discs, ADAFs, and NDAFs, respectively. The region with cross symbols represents photon-cooled discs (cooling dominated either by photon radiation or by advection of photon or gas energy), whereas the region with dot symbols represents neutrino-cooled discs (cooling dominated either by neutrino radiation or by advection of neutrino energy). The figure clearly indicates that, for comparable s, a neutrino-cooled disc generally has a larger β than a photon-cooled disc. In other words, β in a GRB system is generally larger than that in a BHB system.

2.3 A simple analysis

The dependence of β on s can be well understood with equations (5) and (12). Following these two equations, the amplitude of $\delta\dot{M}(r)$, which is the accretion rate variation produced at a radius r, is proportional to the accretion rate at that radius, i.e., $\delta\dot{M}(r) \propto \dot{M}(r)$. Since variations of the accretion rate in the inner region may be represented as a sum of independent accretion rate variations produced at different radii (Lyubarskii 1997), the power spectrum for the flow with $\dot{M} \propto r^s$ can be written as

$$p(f) \propto \dot{M}^2 f^{-1} \propto r^{2s} f^{-1}, \tag{21}$$

where $\dot{M}$ is the accretion rate at the radius r which contributes to the fluctuation at the frequency f, and the term f^{-1} is taken from Lyubarskii (1997) for the constant accretion rate case.

Taking ADAFs as an example, we have $f \propto 1/t_{\rm vis}$ and $t_{\rm vis} = (r/H)^2/(\alpha\Omega) \propto r^{3/2}$, i.e., $r \propto f^{-2/3}$. Equation (21) is therefore simplified as

$$p(f) \propto f^{-(1+\frac{4}{3}s)},$$

which is exactly the same form as equation (16). For other disc models, such as standard thin discs and NDAFs, the results can be understood in the same way.

2.4 Possible influence of the outflow-inflow coupling

As mentioned in §2.1, our results are based on the assumption that the coupling between the local outflow and inflow is weak on the accretion rate fluctuations, i.e., the response of outflows to the inflow fluctuations is weak. In principle, the outflow should have fluctuations related to the variations of the local inflow accretion rate. However, the mechanism for outflows may be complicated, and therefore the fluctuations of the outflow may not be determined simply by that of the local inflow. For example, radiation from the inner region of a disc can heat up the materials in the outer region and thus outflows can be produced (e.g., Begelman et al. 1983; Metzger et al. 2008). In such case the outflow may be more relevant to the inner disc rather than the outer region where it occurs.

Here, we make a simple discussion on the possible influence of the local outflow-inflow coupling as follows. If the coupling exists, the amplitude of accretion rate fluctuation may vary while it propagates into the inner region. With the assumption that $\delta\dot{M}(r)|_{r_1}$ is the accretion rate variation at the radius r_1 induced by $\delta\dot{M}(r)$, we introduce a factor $D(r, r_1)$ to generally describe the change of the above amplitude owing to the coupling effects, i.e., $\delta\dot{M}(r)|_{r_1} = D(r, r_1)\delta\dot{M}(r)$. It is easy to find the form of $D(r, r_1)$ in the following two situations: [1], if the coupling is negligible, the amplitude of $\delta\dot{M}(r)$ will keep unchanged during its propagation, i.e., $\delta\dot{M}(r)|_{r_1} = \delta\dot{M}(r)$, thus $D(r, r_1) = 1$; [2], if the local outflow is in strong coupling with the local inflow, the relative amplitude $\delta\dot{M}(r)|_{r_1}/\dot{M}(r_1)$ will keep unchanged for varying r_1, i.e., $D(r, r_1) = \dot{M}(r_1)/\dot{M}(r)$. Obviously, our results are under the former situation. On the contrary, for the latter one, the power spectrum of accretion rate variations at the inner radius $r_{\rm in}$ can be expressed as

$$p(f) \propto D(r, r_{\rm in})^2 \dot{M}^2 f^{-1} = \dot{M}_{\rm in}^2 f^{-1} \propto f^{-1},$$

which is the same form as the result with a constant accretion rate. (e.g., Lyubarskii 1997). The detailed prescription of $D(r, r_1)$ is, however, beyond the scope of the present work due to the complexity of outflows. Nevertheless, we can expect that a real flow may exist between situations [1] and [2], and therefore the value of β may be located between unity and the results presented in §2.2.

3 SUMMARY AND DISCUSSION

In this paper, we evaluate the effects of outflows on the luminosity fluctuations with the Lyubarskii's general scheme. With a radius-dependent accretion rate $\dot{M} \propto r^s$, the power spectrum of the luminosity fluctuations is $p(f) \propto f^{-\beta}$, where the value of β varies with the disc structure. By assuming that the coupling between the local outflow and inflow is weak on the accretion rate fluctuations, we obtain the following explicit expressions of β for different disc models: $\beta = 1 + 4s/3$ for advection-dominated discs, $\beta = 1 + 40s/(25 - 6s)$ for the outer region of standard thin discs, $\beta = 1 + 10s/(7 - 2s)$ for the middle region, $\beta = 1 + 4s/(7 - 4s)$ for the inner region, and $\beta = 1 + 5s/2$ for NDAFs. The above expressions imply that β in a GRB is generally larger than that in a BHB for comparable s. The expressions of β indicate the possibility of evaluating the strength of outflows by the power spectrum in X-ray binaries and GRBs. In addition, if the coupling is not negligible, the value of β will probably be located between unity and the value presented in the above expressions.

In both BHBs and AGN, ADAFs are usually adopted to describe the quiescent state, the low/hard state, and the corona which lies above a cold disc. ADAFs may produce significant outflows, and therefore the power spectrum can deviate from f^{-1} based on the present analysis. The exact value of s is, however, difficult to estimate from the theoretical point of view, except for the general constraint $0 < s < 1$ (Narayan & McClintock 2008). On the other hand, some observations indicate $s \sim 0.3$ (Yuan et al. 2003; Zhang et al. 2010). Taking this value, we obtain $\beta = 1.4$ for ADAFs, which is close to 1.3, the power-law index of PSDs presented in the low mass X-ray binary systems (Gilfanov & Arefiev 2005). The quantitative difference may be relevant to the coupling between the outflow and inflow as discussed in §2.4.

If there is only outflow that operates in the accreting system, s should be positive. However, s can also be negative due to the evaporation mechanism of a cold disc. For the accreting black hole in BHBs, the observed power-law components in the X-ray spectra are generally attributed to hot, tenuous plasmas, namely accretion disc coronae. Due to the high temperature in the corona, the interaction between the disc and corona would lead to mass evaporating from the disc to the corona (Meyer et al. 2000; Spruit & Deufel 2002). In this case, the value of s for the corona should be negative if outflows are not strong, and therefore it is quite possible for β to be less than unity. Consequently, in this scenario s for the underneath cold disc should be positive.

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APPENDIX A: THE VALIDITY OF THE ASSUMPTION OF CONSTANT Q

In this section, we analyze the validity of the assumption of constant Q.

With a radius-dependent accretion rate described in equation (10), we have

$$\Phi = s\frac{\dot{M}_{0,\rm in}}{r_{\rm in}}(\frac{r}{r_{\rm in}})^{(s-1)},$$

then equation (4) can be expressed as

$$Q = 1 - \frac{\dot{M}_{\rm in} l_{\rm in} + s\int_{r_{\rm in}}^{r} \frac{\dot{M}_{0,\rm in}}{r_{\rm in}}(\frac{r}{r_{\rm in}})^{(s-1)} l_\Phi dr}{\dot{M}\Omega r^2}.$$

We assume that the specific angular momentum corresponding to Φ is proportional to that of the gas in the disc, i.e.

$$l_\Phi \propto \Omega r^2 \propto r^{1/2}.$$

This is the case for thermal energy driving outflows, magnetic field centrifugal accelerating wind (Mayer & Pringle 2006) and disc evaporation model.

For the general situation ($s \neq -1/2$), we have

$$Q = 1 - \frac{\dot{M}_{\rm in} l_{\rm in} + \frac{s}{s+0.5}\left[\dot{M} l_\Phi\right]_{r_{\rm in}}^{r}}{\dot{M}\Omega r^2}$$
$$= 1 - \frac{s}{s+0.5}\frac{l_\Phi}{\Omega r^2} - (l_{\rm in} - \frac{s}{s+0.5} l_{\Phi,\rm in})(\frac{r_{\rm in}}{r})^s \frac{1}{\Omega r^2},$$

and therefore

$$Q \to \text{const.} \quad \text{for} \quad s > -1/2,$$

where $l_{\Phi,\rm in}$ is the angular momentum of Φ at the inner radius $r_{\rm in}$. Thus, the analysis presented in this paper holds for $s > -1/2$. For $s < -1/2$, our analysis may present a qualitative result.

APPENDIX B: FLICKER NOISE IN ADAFS

The dynamic equations of ADAFs read as follows (Narayan & Yi 1994; Kato et al. 2008; Li & Cao 2009),

$$\frac{\partial}{\partial t}(2H\rho) = \frac{1}{2\pi r}\left(\frac{\partial \dot{M}}{\partial r} - \Phi\right),$$

$$\rho r\Omega^2 - \rho r\Omega_{\rm K}^2 - \frac{\partial p}{\partial r} = 0,$$

$$2H\rho\left[\frac{\partial v_\varphi}{\partial t} + \frac{v_r}{r}\frac{\partial}{\partial r}(r v_\varphi)\right] + \frac{\Phi}{2\pi r^2}\Delta l = \frac{1}{r^2}\frac{\partial}{\partial r}\left(2Hr^2\tau_{r\varphi}\right),$$

$$\frac{1}{\gamma_2 - 1}\left(\frac{\partial p}{\partial t} + v_r\frac{\partial p}{\partial r} - \gamma_1\frac{p}{\rho}\frac{\partial \rho}{\partial t} - \gamma_1\frac{p}{\rho}v_r\frac{\partial \rho}{\partial r}\right) = -\tau_{r\varphi}\left(r\frac{\partial \Omega}{\partial r}\right),$$

where γ_1 and γ_2 are the usual generalized ratios of the specific heat , $\tau_{r\varphi} = -\alpha p$, $H/r = $ const., and Φ, $\Delta l = l_\Phi - r v_\varphi$ maintain the value of the stationary state. The self-similar solution of the above equations is

$$v_{r,0} \propto r^{-1/2},\ \Omega_0 \propto r^{-3/2},\ p_0 \propto r^{-5/2+s},\ \rho_0 \propto r^{-3/2+s}.$$

We introduce small deviations of the disc parameters from the stationary parameters as follows,

$$\alpha = \alpha_0(1+\overline{\beta}), \quad \overline{\beta}' = r^s\overline{\beta},$$
$$v_r = -a_1\alpha_0 r\Omega_{\rm K}\left(1 + r^{-s}\upsilon\right), \quad \Omega = a_2\Omega_{\rm K}\left(1 + r^{-s}\omega\right),$$
$$p = a_3\sqrt{r}\Omega_{\rm K}^2 r^s\left(1 + r^{-s}\delta\right), \quad \rho = a_4 r^{-3/2+s}\left(1 + r^{-s}\sigma\right).$$

where a_1, a_2, a_3, and a_4 are constant. With the above equations, the evolution equations of the perturbed variables are

$$\frac{1}{\alpha_0\Omega_{\rm K}}\frac{\partial \sigma}{\partial t} = -a_1 r\frac{\partial(\upsilon+\sigma)}{\partial r},$$

$$a_3 r\frac{d\delta}{dr} - \frac{5}{2}a_3\delta - 2a_4a_2^2\omega - a_4\left(a_2^2 - 1\right)\sigma = 0,$$

$$\frac{1}{\alpha\Omega_{\rm K}}\frac{\partial \omega}{\partial t} - a_1 r\frac{\partial \omega}{\partial r} + \frac{a_3}{a_4a_2}r\frac{\partial \delta}{\partial r} - \frac{1}{2}a_1(1-2s)\omega - \frac{1}{2}a_1\upsilon -$$
$$\frac{1}{2}a_1\sigma + \frac{a_3}{2a_4a_2}\delta = \frac{a_3}{a_4a_2}r\frac{\partial\overline{\beta}'}{\partial r} + \frac{a_3}{2a_4a_2}\overline{\beta}',$$

$$\frac{1}{\alpha\Omega_{\rm K}}\frac{\partial \delta}{\partial t} - \gamma_1\frac{1}{\alpha\Omega_{\rm K}}\frac{d\sigma}{dt} - a_1 r\frac{\partial \delta}{\partial r} + \gamma_1 a_1 r\frac{\partial \sigma}{\partial r} - (\gamma_2 - 1)\times$$
$$a_2 r\frac{\partial \omega}{\partial r} + \left[\frac{5}{2}a_1 + \left(s - \frac{3}{2}\right)\gamma_1 a_1 + \frac{3}{2}(\gamma_2 - 1)a_2\right]\delta +$$
$$\left(\frac{5}{2} - s - \frac{3}{2}\gamma_1 + s\gamma_1\right)a_1\upsilon - s\gamma_1 a_1\sigma + \left(s + \frac{3}{2}\right)\times$$
$$(\gamma_2 - 1)a_2\omega = -\frac{3}{2}(\gamma_2 - 1)a_2\beta'.$$

The accretion rate is

$$\dot{M} = -4\pi r v_r\rho H,$$

and its fluctuating component is

$$\dot{m}(r,f) = 4\pi a_1a_4\alpha_0\left(\frac{H}{r}\right)[\upsilon + \sigma].$$

For $\sqrt{\langle\bar{\beta}\rangle^2} \propto r^0$, the power spectrum is (see Lyubarskii 1997, Section 5)

$$p(f) \propto f^{-(1+\frac{4}{3}s)}.$$